



Evolution of Asteroid Orbits in a Restricted Three-Body Simulation

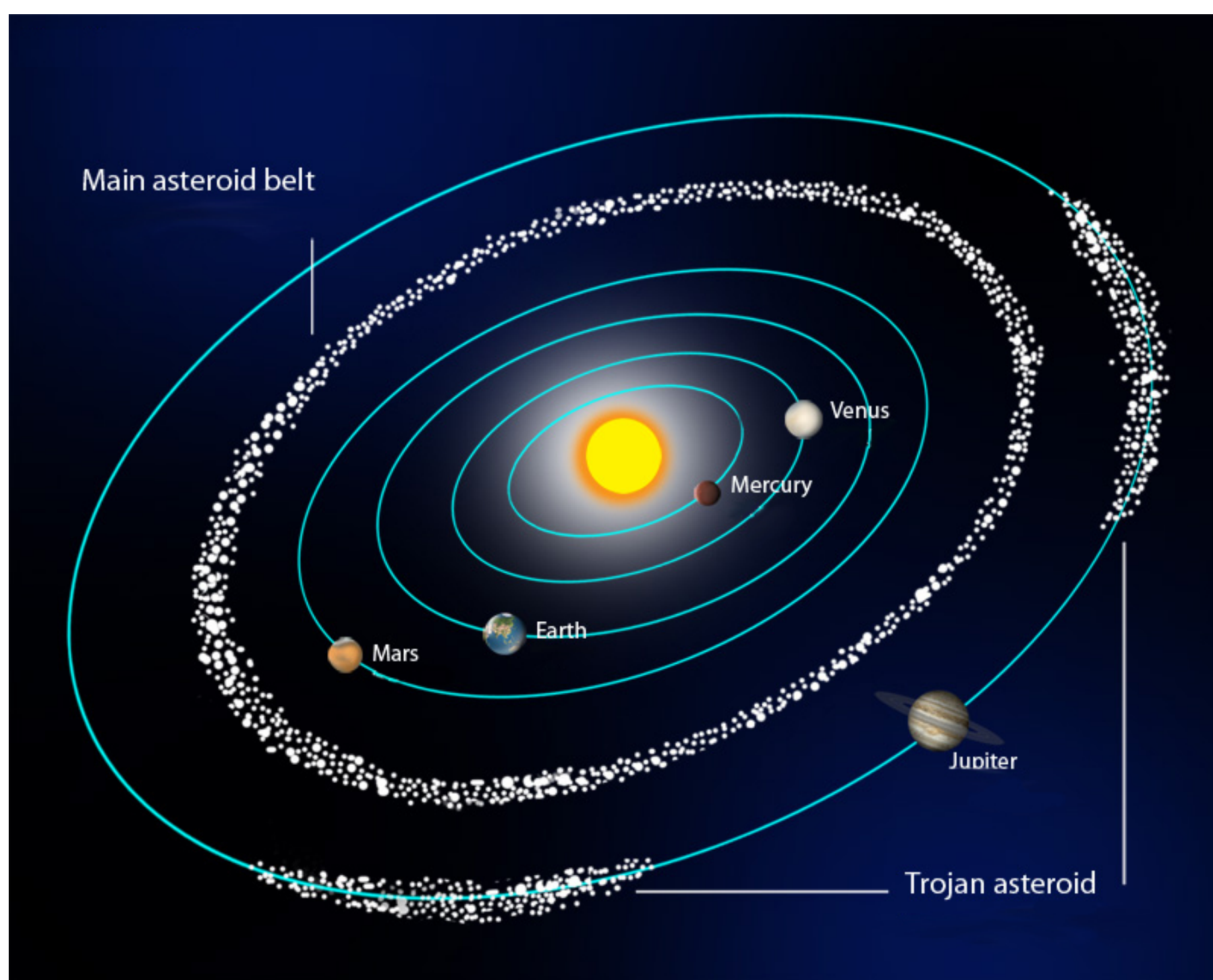
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Abstract

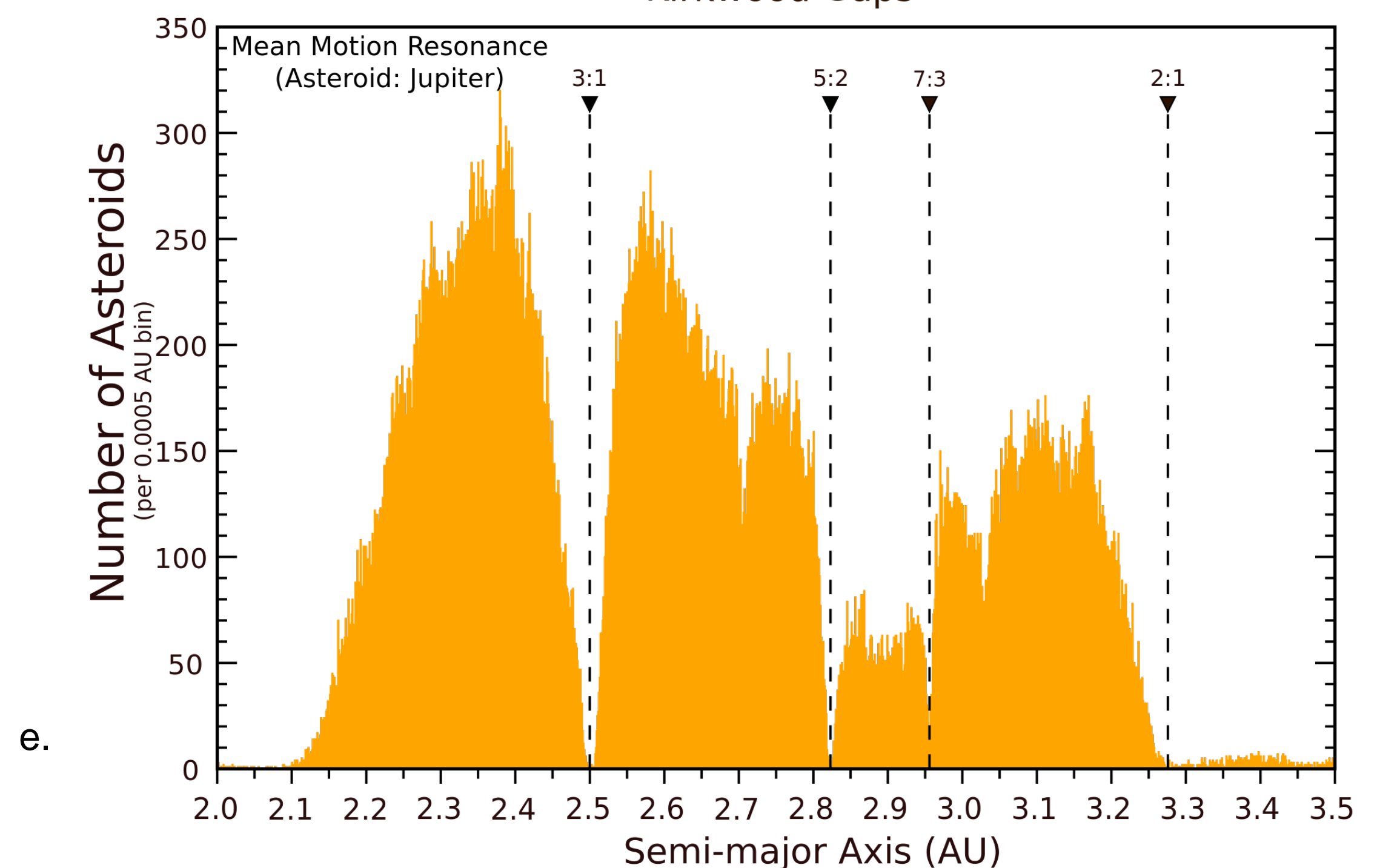
We study the evolution of asteroid orbits in a restricted three-body problem formulation consisting of the Sun, the planet Jupiter and an unspecified asteroid of negligible mass. It was discovered by Kirkwood [1] that the distribution of asteroid orbits contains gaps for orbits whose period is commensurate with that of Jupiter. Detailed computations in three-dimensional, many-body formulations found that test bodies initially placed in a forbidden orbit did not develop large eccentricities or leave the gap even after the passage of 10^5 years [2]. In the present two-dimensional simulation, an extension of earlier work [3], we perform numerical integrations of the coupled equations of motion for Jupiter and the asteroid. Under assumptions of a stationary Sun and a circular orbit for Jupiter, we find that test bodies initially placed in a forbidden orbit can develop a large eccentricity after relatively few orbits.

Introduction

Asteroids, also known as minor planets or planetoids, are rocky bodies ranging in size from dust particles to 1,000 km across. While most are contained in stable orbits in a belt located between the orbits of Jupiter and Mars, some have highly eccentric orbits and can cross the plane of Earth's orbit. The figure on the left depicts the location of the Asteroid Main Belt in the Solar System. The figure on the right shows the distribution of asteroid orbits, specifically the distribution of semi-major axes of the orbits. Gaps in the distribution occur at positions where an orbit semi-major axis is an integer fraction of that of Jupiter. These are known as the Kirkwood gaps and the absence of asteroids in such orbits is attributed to resonant phenomena associated with the gravitational influence of Jupiter.

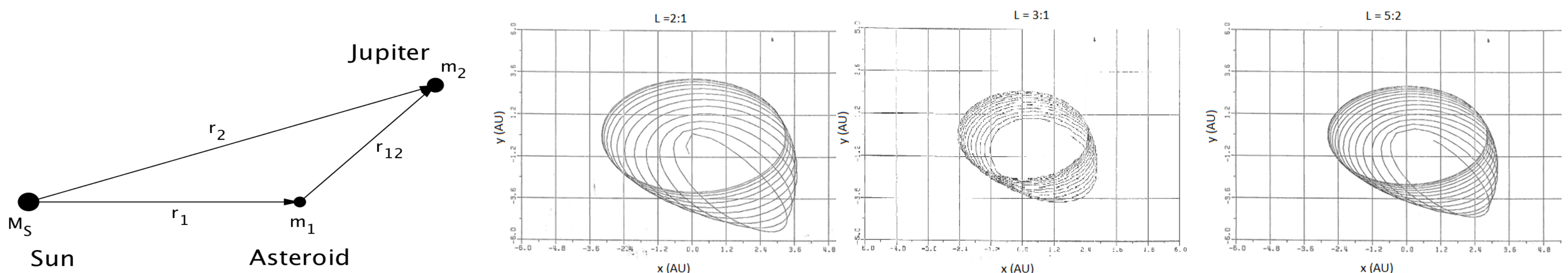


Asteroid Main-Belt Distribution
Kirkwood Gaps



Present Work

In a two dimensional, three-body formulation we study the evolution of orbits of bodies with initial location at forbidden values of the semi-major axis. The exact problem is a three-dimensional many-body problem requiring solution of coupled differential equations equal in number to three times the number of gravitationally interacting bodies. With only the Sun, Jupiter and an asteroid in a planar configuration, the number of coupled differential equations is reduced to four. They are further simplified by neglect of the effect of asteroid mass upon the motion of Jupiter. The configuration and graphs of the computed orbital evolution after ten orbits for three cases of forbidden locations are shown below.



Summary and Conclusions

The computed orbits shown are for test bodies placed in forbidden locations corresponding to Jupiter-to-asteroid semi-major axis ratios of 2:1, 3:1 and 5:2. Whereas many-body calculations had shown such orbits to be stable for more than 10^5 years, in the present case significant departures developed in fewer than ten revolutions.

References

1. D. Kirkwood, Proc. AAAS, pp.8-14 (1866)
2. See, for exsmple, J.Wisdom, AJ, **87**, 577 (1982).
3. D. W. Kraft, Bull. Am. Phys. Soc **33**, 64 (1988).